Chapter 4
Section 4.5
Word Problems

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1. A company manufactures baseball caps with school logos. The company charges their customers a fixed fee of \$500 for setting up the machines and \$8 for each cap produced.

(a) Find a linear model for the cost of purchasing any number of caps from this company. Express this model as a function.

(b) Use the model to find the cost of purchasing 225 caps.

(c) Use the model to find the number of caps sold if the customer pays \$2100.

(a) $C(x) = 8x + 500$

(b) $C(225) = 8(225) + 500 = \$2,300$

(c) $C(x) = 2100 = 8x + 500 \Rightarrow 1600 = 8x \Rightarrow x = 200$ caps

2. The average U.S. resident uses 650 lb of paper a year. The average pine tree produces 4130 lb of paper.

(a) Find a function N that models the number of trees used for paper in one year by x U.S. residents.

(b) The city of Cleveland Heights, Ohio, had a population of 49,000 in 2003. Use the model to find the number of trees required to make the paper used by the residents of Cleveland Heights in 2003.

(c) Use the model to determine how many U.S. residents can be supplied with paper for a year from 200 pine trees.

(a) $N(x) = \frac{650}{4130} x$

(b) $N(49{,}000) = \frac{650}{4130} \cdot 49{,}000 = 7{,}711.86$ trees

(c) $N(x) = 200 = \frac{650}{4130} x \Rightarrow x = 200 \cdot \frac{4130}{650} = 1{,}270.77$ people.

3. A gardener waters his vegetble plot using a drip irrigation system. Water flows slowly from a 1200-gallon tank through a perforated hose network to keep the soil appropriately moist. During the spring planting season, the garden requires 80 gallons of water per day.

(a) Find a function W that gives the amount of water in the tank x days after it has been filled.

(b) Use the function W to find the water remaining in the tank after 3 days and after 12 days.

(c) Calculate $W(20)$. What does this answer tell you?

(d) How many days will it take for the tank to empty?

(e) The gardener prefers not to let the tank empty completely. Instead, he decides to refill it when the water level has dropped to 200 gallons. How many days should he wait to refill the tank and how many gallons of water will he need?

(a) $W(x) = 1200 - 80x$

(b) $W(3) = 1200 - 80(3) = 960$ gallons remaining after 3 days

$W(12) = 1200 - 80(12) = 240$ gallons remaining after 12 days.

(c) $W(20) = 1200 - 80(20) = -400$ gallons, so the tank has run out of water.

(d) $0 = W(x) = 1200 - 80x \Rightarrow 1200 = 80x \Rightarrow x = 15$ days.

(e) $200 = W(x) = 1200 - 80x \Rightarrow 80x = 1000 \Rightarrow x = 12.5$ days and he needs 1000 gallons.

4. A breakfast cereal company manufactures boxes to package their product. For aesthetic reasons, the box must have the following proportions: Its width is 3 times its depth, and its height is 5 times its depth.

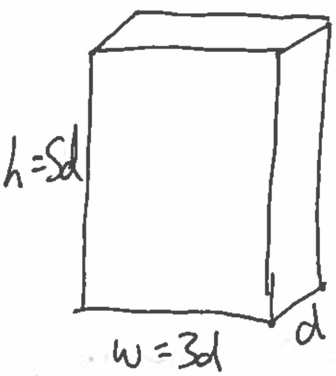
(a) Why might a cereal company have aesthetic reasons for the dimensions of its box?

(b) Find a function that models the volume of the box in terms of its depth, and graph the function.

(c) Find the volume of the box if the depth is 1.5 in.

(d) For what depth is the volume 90 in^3?

(e) For what depth is the volume greater than 60 in^3?



(a) Advertising/Sales

(b) $V = h \cdot w \cdot d = 5d \cdot 3d \cdot d = 15d^3$

(c) $V(1.5) = 15(1.5)^3 = 50.625 \text{ in}^3$

(d) $90 = V(d) = 15d^3 \Rightarrow d^3 = 6 \Rightarrow d = \sqrt[3]{6} =$

$= 1.82$ in.

(e) $60 = V(d) \geq 15d^3$

$4 \geq d^3$

$d \leq \sqrt[3]{4} \approx 1.59$ in.

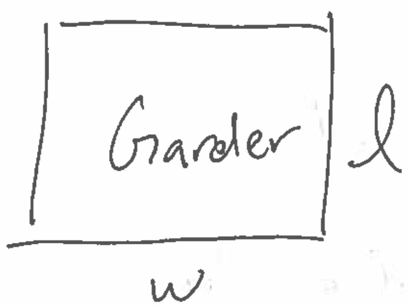
5. A gardener has 140 ft of fencing to fence a rectangular vegetable garden.

(a) Find a function that models the area of the garden she can fence with respect to the width.

(b) For what range of widths is the area greater than 825 ft^2?

(c) Can she fence a garden with area 1250 ft^2?

(d) Find the dimensions of the largest area she can fence.



(a) $140 = 2l + 2w$

$140 - 2w = 2l$

$70 - w = l$

$A = l \cdot w = (70 - w) \cdot w = 70w - w^2$

(b) $825 = A(w) \geq 70w - w^2$

$0 \geq -w^2 + 70w - 825$

$0 \leq w^2 - 70w + 825 = (w-15)(w-55)$

width is between 15 and 55 ft

$w \in [15, 55]$

(c) $1250 = A(w) = 70w - w^2$

$0 = w^2 - 70w + 1250$ $\quad \sqrt{70^2 - 4 \cdot 1 \cdot 1250} = \sqrt{-100}$ so no.

(d) Area maximized at $w = \frac{-b}{2a} = \frac{70}{2} = 35$ ft.

So max area $= 35 \cdot 35 = 1{,}225$ ft^2